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First principle derivation of semiclassical force for electroweak baryogenesis

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Abstract

We perform a systematic gradient expansion on kinetic equations and derive the CP-violating semiclassical force for fermions propagating in presence of a CP-violating wall at a first order electroweak phase transition. The force appears at order $\hbar$ in the flow term of the kinetic equation and agrees with the semiclassical force used for baryogenesis computations. In particular we consider the force for charginos in both the MSSM and NMSSM. We then study the continuity equations for the vector and axial vector currents and stress the role of the latter as the one containing baryogenesis sources. We also show that there is no CP-violating force for bosons to order $\hbar$ in gradient expansion.

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1 Introduction

The creation of a baryon asymmetry at a first-order electroweak phase transition in the early universe is an attractive proposal [1] because the elementary particles and interactions involved in this process can be tested soon in accelerator experiments. For a successful baryogenesis a coalition between CP violation, nonequilibrium thermodynamics and baryon number violation is needed [2]. Model calculations require a study of generation and transport of CP-violating flows arising from interactions of fermions with the expanding phase transition fronts. As the problem involves the dynamics of quantum fields in a spatially varying background it cannot be treated within the classical transport theory. While fully general quantum Boltzmann equations can quite easily be formulated by making use of suitably truncated Dyson-Schwinger equations for the out-of-equilibrium two-point functions, some approximation scheme is needed to derive a set of practically solvable, yet sufficiently general transport equations for electroweak baryogenesis (EWBG).

A fast baryon number violating rate in the unbroken phase is a necessary ingredient of any EWBG model. However, to avoid a wash-out of the newly created asymmetry, the baryon number violation must turn off in the Higgs phase. As is well known, this is the case provided the transition is strong enough [3]. Since for the present experimental bounds on the Higgs mass the electroweak phase transition in the Standard Model (SM) is not first-order [4], one is lead to consider extensions of the Standard Model. The natural candidates are supersymmetric models, which include the Minimal Supersymmetric Standard Model (MSSM) [5] and the nonminimal extension (NMSSM) [6] with an additional Higgs-singlet field. These models contain additional scalars which can strengthen the phase transition as required for baryogenesis.

In supersymmetric models the bubble walls are found to be quite slow and thick [7, 8, 9, 10, 11], in the sense that $v_{\rm wall} \ll c$ and $\ell_{\rm wall} \gg \ell_{\rm dB}$, where $\ell_{\rm dB} \sim 1/T$ denotes a typical de Broglie wave length of thermal particles. The latter condition is of particular importance, because it implies that a gradient expansion in terms of $\ell_{\rm dB}/\ell_{\rm wall}$ represents a controlled, rapidly converging approximation scheme for most of the excitations in the electroweak plasma.

In the past many heuristic attempts have been made to derive approximate transport equations for EWBG [12, 13, 14, 15, 16, 17]. Common to all these methods is the strategy to somehow isolate the essential quantum features of the transport in the form of "sources" to be inserted into classical transport equations. Baryon production has in this way been computed in two doublet models [13, 14, 15, 18], MSSM [16, 19, 20, 21, 22, 23] and NMSSM [24, 25]. Different approaches, when applied to the same physical problem, have been found to disagree however. In particular the sources from chargino and squark sectors in the MSSM, found using WKB-methods [21, 22, 24], are parametrically different from those derived by the use of the continuity equations and the relaxation time approximation [19, 20], and by other earlier attempts [16, 26].

In this paper we present a rigorous first-principle derivation of quantum transport equations appropriate for baryogenesis in the limit of thick phase boundaries ignoring collisions. We start our analysis by writing the exact Dirac equation of motion for the dynamical Green function (Wightman function) $G^{<}_{\alpha\beta}(u,v) \equiv i\langle\bar{\psi}_\beta(v)\psi_\alpha(u)\rangle$ with a CP-violating spatially varying pseudoscalar mass term. For simplicity here we consider particles moving perpendicular to the phase boundary, which effectively reduces our problem to 1+1 dimensions. The results discussed here are not affected in any important way when the general 3+1 dimensional case is considered [27]. By performing a Wigner transform we obtain a controlled expansion in gradients, or more appropriately, in powers of $\hbar$. We show that, to first order in $\hbar$, $G^{<}$ admits a spectral decomposition in terms of on-shell quasiparticle excitations. The on-shell momenta are set by a dispersion relation derived from the equations of motion and agree with the results of [28], where the spectral function $\mathcal{A}$ was considered in gradient expansion. The on-shell distribution functions f_{s+} and f_{s-} for particles and antiparticles of spin s, respectively, are then shown to obey the following kinetic Liouville equations:

$$\partial_t f_{s\pm} + v_{s\pm}\partial_z f_{s\pm} + F_{s\pm}\partial_{k_z} f_{s\pm} = 0. \tag{1}$$

The quantum information in (1) is entirely contained in the expression for the quasiparticle energy $\omega_{s\pm}$, which shows up in the expressions for the *group velocity* $v_{s\pm} \equiv k_z/\omega_{s\pm}$ and the *semiclassical force* $F_{s\pm} = \omega_{s\pm} dv_{s\pm}/dt$, where k_z denotes the kinetic momentum. For example, we show that in the case of a single chiral fermion moving in a CP-violating background

with planar symmetry, represented by a z-dependent complex mass $m(z) = |m(z)|\mathrm{e}^{i\theta(z)}$, a quasiparticle moving in z-direction with momentum k_z has the energy

$$\omega_{s\pm} = \omega_0 \mp \hbar \frac{s|m|^2\theta'}{2\omega_0^2}\,, \tag{2}$$

where $\omega_0 \equiv \sqrt{k_z^2 + |m|^2}$, and experiences the force

$$F_{s\pm} = -\frac{|m|^{2\,\prime}}{2\omega_{s\pm}} \pm \hbar \frac{s(|m|^2\theta')'}{2\omega_0^2}\,. \tag{3}$$

We also derive explicit expressions for the semiclassical force for a general case of N mixing fermions and in particular for charginos both in the MSSM and NMSSM. We then show that in the case of N mixing bosonic fields, such as the squarks in the (N)MSSM, there is no CP-violating force to first order in $\hbar$.

Our results agree with recent results obtained by the use of the WKB-approach [22, 24]. The WKB-method was originally introduced by Joyce, Prokopec and Turok in [29] and [15] and then applied to the MSSM in [21]. The CP-violating velocities and accelerations for fermions interacting with a phase transition wall were correctly computed from the WKB-dispersion relations in Refs. [29, 15, 21]. The velocity and force in kinetic equations were obtained from the Hamilton equations based on canonical momentum. However, when the dispersion relation is derived by considering the spectral function in gradient approximation [28], the momentum appearing in the Wigner representation is the kinetic momentum. The relevance of the kinetic momentum as the true physical variable in the WKB-picture was first realized by Cline, Joyce and Kainulainen [22], who also showed how it can be consistently incorporated into kinetic theory leading to equations identical to (1-3).

The outstanding contribution of the present work is a controlled first principle derivation of the kinetic equation (1). This is important because of a considerable controversy in literature concerning the transport equations to be used for EWBG calculations. Moreover, our treatment in principle allows a study of the plasma dynamics beyond first order in $\hbar$, which cannot be achieved by WKB-methods. As an example, at second order in $\hbar$ the full equations do not admit the spectral decomposition solution for $G^<$; for a scalar field this problem is considered in [30].

Let us mention that in a related work [31] the Liouville equations for fermions in presence of a classical gauge field have been considered in gradient approximation. The crucial role of the constraint equations in the derivation of the kinetic equations was then stressed in [32, 33]. The problem of a pseudo-scalar mass term in kinetic equations has been considered in Refs. [34, 32], but these authors discussed the flow term only to zeroth (classical) order in $\hbar$, whereas the spin dependent force essential for EWBG arises only at quantum level, as we show here.

Inclusion of interaction terms gives rise to yet another source which is of first order in $\hbar$, namely the *spontaneous baryogenesis* (SBG) source of Ref. [35]. The SBG source appears because the CP-violating split in the dispersion relation causes the CP-conjugate states to relax towards different local equilibria in the bubble wall. Thus a first principle derivation of the SBG source requires not only a consistent expansion in $\hbar$, which we have done, but also a consistent expansion in relevant coupling constants. The latter is necessarily a model dependent problem and shall be considered elsewhere. However, to facilitate comparison with literature we derive the vector current to order $\hbar$ in gradient expansion which displays the SBG source in the relaxation time approximation. When applied to the MSSM, our results differ from Refs. [36], [17] and [20].

The paper is organized as follows. In section 2 we derive the Liouville equations for a single Dirac fermion with a spatially varying complex mass term. In section 3 we generalize these results to the case of N mixing fermionic fields and study the case of mixing charginos in both the MSSM and NMSSM. We then in section 4 consider the case of N mixing scalar fields and show that there is no CP-violating source to first order in $\hbar$. In section 5 we study the continuity equations for both vector and axial vector current, and spontaneous baryogenesis in the relaxation time approximation, and make a comparison with literature. For example, in contrast to what is claimed in [37] and [17], we find that the continuity equation for the vector current contains no CP-violating source in the absence of collisions. On the other hand, in the continuity equation for the axial current there are CP-violating sources that can be related to higher moments expansion of the semiclassical Boltzmann equation. Finally, section 6 contains a discussion and summary.

2 Fermionic field with a complex mass

We first consider the dynamics of a fermionic field with a complex spatially varying mass term. More precisely, we take our system to be described by the effective lagrangian

$$\mathcal{L} = i\bar{\psi}\,\partial\!\!\!/\,\psi - \bar{\psi}_L m \psi_R - \bar{\psi}_R m^* \psi_L + \mathcal{L}_{\rm int}\,, \tag{4}$$

where $\mathcal{L}_{\rm int}$ contains interactions and

$$m(x) = m_R(x) + i m_I(x) = |m(x)|{\rm e}^{i\theta(x)} \tag{5}$$

is a space-time dependent mass term arising from an interaction with some CP-violating scalar field condensate. We are primarily interested in the case where m arises from the Higgs field condensate at a first order electroweak phase transition. As the bubbles of broken phase grow several orders of magnitude larger than the wall width before coalescence, the wall can be approximated by a planar interface to good accuracy. We therefore consider a mass term in the bubble wall frame which is only a function of the spatial coordinate orthogonal to the wall, $m = m(z)$.

Our focus in this paper is on the semiclassical treatment of fermions in presence of background fields. In particular we derive the flow term for a fermionic kinetic equation with a nontrivial force induced by the CP-violating mass parameter (5). To keep the discussion simple, we relegate the explicit treatment of collision terms and self-energy corrections which are induced by the specific interactions included in $\mathcal{L}_{\rm int}$, to later publications [27]. Moreover, we are interested in the case of wide walls, so that the de Broglie wave length $\ell_{\rm dB}$ of a typical excitation is small in comparison with the wall width, $\ell_{\rm dB} \ll \ell_{\rm wall}$. This condition is amply satisfied at the electroweak phase transition, where typically $\ell_{\rm dB} \sim 1/T$ and $\ell_{\rm wall} \sim 10/T$ [8].

With the above assumptions we now develop the equations of motion for the Wightman function

$$G^<_{\alpha\beta}(u,v) \equiv i\left\langle \bar{\psi}_\beta(v)\psi_\alpha(u)\right\rangle \tag{6}$$

in a consistent expansion in gradients of the background fields. Here $\langle\cdot\rangle$ denotes the expectation value with respect to the initial state. The function $G^<$ describes the statistical properties of an out-of-equilibrium system. It corresponds to the off-diagonal part of the

fermionic two-point function in the Schwinger-Keldysh formalism [38], which indeed is the method of choice to derive the equations of motion for $G^<$ including interactions. However, in their absence all one needs is the familiar Dirac equation. Dropping interactions we have from Eq. (4):

$$\left(i\,\partial\!\!\!/_u - m_R(u) - i\gamma^5 m_I(u)\right)\psi(u) = 0\,. \tag{7}$$

Multiplying (7) from the left by the spinor $i\bar{\psi}(v)$ and taking the expectation value one finds:

$$\left(i\,\partial\!\!\!/_u - m_R(u) - i\gamma^5 m_I(u)\right)G^<(u,v) = 0\,. \tag{8}$$

This equation and the hermiticity property

$$\left[i\gamma^0 G^<(u,v)\right]^\dagger = i\gamma^0 G^<(v,u), \tag{9}$$

which can be immediately inferred from the definition (6), completely specifies $G^<$.

In order to study equation (8) in gradient expansion we perform a Wigner transform of $G^<$ to the mixed representation, *i.e.* a Fourier transform with respect to the relative coordinate $r \equiv u - v$:

$$G^<(x,k) \equiv \int d^4 r\, e^{ik\cdot r} G^<(x + r/2, x - r/2), \tag{10}$$

where $x = (u+v)/2$ denotes the center-of-mass coordinate. The crucial advantage of the representation (10) is that it separates the internal fluctuation scales, described by momenta k, from the external ones which show up as a dependence of $G^<$ on x, and thus gives us the chance of exploiting possible hierarchies between these scales. In the Wigner representation equation (8) becomes

$$\left(\hat{k\!\!\!/} - \hat{m}_0(x) - i\hat{m}_5(x)\gamma^5\right)G^< = 0, \tag{11}$$

where we use the following convenient shorthand notation

$$\hat{k}_\mu \equiv k_\mu + \frac{i}{2}\partial_\mu \tag{12}$$

$$\hat{m}_{0(5)} \equiv m_{R(I)} e^{-\frac{i}{2}\overleftarrow{\partial}_x\cdot\,\partial_k}. \tag{13}$$

The original local equation (8) for $G^<$ is thus transformed into an equation involving an infinite series in gradients. This in fact can be viewed as an expansion in powers of the

Planck constant $\hbar$. We have set $\hbar \to 1$, but a dimensionful $\hbar$ can at any stage be easily restored by the simple replacements $\partial_x \to \hbar\,\partial_x$ and $G^< \to \hbar^{-1}\,G^<$.

Because of the planar symmetry (here we do not consider initial states of the plasma that break this symmetry) $G^<$ can depend only on the spatial coordinate orthogonal to the wall, $z \equiv x^3$. We also consider equation (11) only in a frame where the momentum parallel to the wall vanishes, $\vec{k}_\parallel = 0$. This last assumption effectively reduces the problem to 1+1 dimensions, and we can cast equation (11) into the form

$$\left(\hat{k}_0 + \hat{k}_z\gamma^0\gamma^3 - \hat{m}_0\gamma^0 + i\hat{m}_5\gamma^0\gamma^5\right) i\gamma^0 G^< = 0, \tag{14}$$

where $\hat{k}_0 = k_0 + \frac{i}{2}\partial_t$ and $\hat{k}_z = k_z - \frac{i}{2}\partial_z$. The differential operator in (14) is entirely spanned by a closed 1+1-dimensional subalgebra of the full 3+1-dimensional Clifford algebra. Moreover, it commutes with the operator $S^3 = \gamma^0\gamma^3\gamma^5$, which measures spin s in z-direction. s is thus a good quantum number in the frame $\vec{k}_\parallel = 0$, which motivates to seek solutions for $i\gamma^0 G^<$ which satisfy $S^3 i\gamma^0 G_s^< = i\gamma^0 G_s^< S^3 = s i\gamma^0 G_s^<$. Working in a convenient chiral representation this condition leads immediately to the following spinor structure:

$$-\,i\gamma^0 G_s^< = \frac{1}{2}(1 + s\sigma^3) \otimes g_s^<, \tag{15}$$

where σ^3 is the Pauli matrix referring to spin in z-direction. $g_s^<$ has indices in the remaining two dimensional chiral space and can also be written in terms of the Pauli matrices ρ^i as follows:

$$g_s^< \equiv \frac{1}{2}\left(g_0^s + g_i^s\rho^i\right). \tag{16}$$

The decomposition (15) contains the implicit assumption that $i\gamma^0 G^<$ does not mix spins. We expect that this is a good approximation in the electroweak plasma. The signs and normalizations in (15-16) are chosen such that g_0^s measures the number density of particles with spin s in phase space. With these simplifications we can now reduce our original 4×4 problem to a two-dimensional one by effecting the replacements

$$\gamma^0 \;\to\; \rho^1, \qquad -i\gamma^0\gamma^5 \;\to\; \rho^2, \qquad -\gamma^5 \;\to\; \rho^3 \tag{17}$$

and we find

$$\left(\hat{k}_0 - s\hat{k}_z\rho^3 - \rho^1\hat{m}_0 - \rho^2\hat{m}_5\right) g_s^< = 0. \tag{18}$$

A similar procedure is used in [28] for a treatment of the fermionic propagator. Equation (18) may look simple, but it still constitutes a set of four coupled complex (or eight coupled real) differential equations, which we shall now analyze. We first find the four independent complex equations for g_a^s $(a = 0, i)$ by multiplying (18) successively by 1 and ρ^i, and taking the trace:

$$\begin{aligned} \hat{k}_0 g_0^s - \; s\hat{k}_z g_3^s - \; \hat{m}_0 g_1^s - \; \hat{m}_5 g_2^s &= 0 &(19)\\ \hat{k}_0 g_3^s - \; s\hat{k}_z g_0^s - i\hat{m}_0 g_2^s + i\hat{m}_5 g_1^s &= 0 &(20)\\ \hat{k}_0 g_1^s + is\hat{k}_z g_2^s - \; \hat{m}_0 g_0^s - i\hat{m}_5 g_3^s &= 0 &(21)\\ \hat{k}_0 g_2^s - is\hat{k}_z g_1^s + i\hat{m}_0 g_3^s - \; \hat{m}_5 g_0^s &= 0. &(22) \end{aligned}$$

As a consequence of Eq. (9) the matrices $g_s^<$ are hermitean so that g_a^s are real functions. We then have twice as many equations as independent functions corresponding to real and imaginary parts of Eqs. (19-22), and hence one half of the equations must correspond to the constraints on the solutions of the other half; those equations are kinetic equations. This was first pointed out in the context of kinetics of fermions by Zhuang and Heinz [32]. As one sees from (12), no time derivatives appear in the real parts of (19-22) so that they indeed provide four *constraint equations* (CE) on the solutions of four *kinetic equations* (KE). These contain time derivatives and correspond to the imaginary parts of (19-22).

Because we have put no restrictions to the form of the $\hat{m}$-operators, Eqs. (19-22) are still valid to any order in gradients in the frame where $\vec{k}_\| = 0$. In what follows we assume that the mass is a slowly varying function of x and truncate gradient expansion at second order, which is the lowest order at which CP-violating effects can be discussed consistently. This method is not adequate for problems involving quantum mechanical reflection, which require nonperturbative treatment in $\hbar$. With the truncation, $\hat{m}_{0(5)}$ in (19-22) simplifies to

$$\hat{m}_{0(5)} \simeq m_{R(I)} + \frac{i}{2} m'_{R(I)} \partial_{k_z} - \frac{1}{8} m''_{R(I)} \partial_{k_z}^2 . \tag{23}$$

Even with this truncation, we are facing a problem involving eight coupled second order partial differential equations. Our next task is to reduce these to a single equation governing the dynamics of the fermionic two-point function.

2.1 Constraint equations

Let us first consider the constraint equations. They consist of four homogeneous equations for four functions, which implies that there is one constraint which gives rise to the dispersion relation. While this property remains true to any order in gradients (or equivalently in $\hbar$), we only need to work to first order to find the nontrivial result we are looking for. To this order we have

$$k_0 g_0^s - s k_z g_3^s - m_R g_1^s - m_I g_2^s = 0 \tag{24}$$

$$k_0 g_3^s - s k_z g_0^s + \frac{1}{2} m_R' \partial_{k_z} g_2^s - \frac{1}{2} m_I' \partial_{k_z} g_1^s = 0 \tag{25}$$

$$k_0 g_1^s + \frac{s}{2} \partial_z g_2^s - m_R g_0^s + \frac{1}{2} m_I' \partial_{k_z} g_3^s = 0 \tag{26}$$

$$k_0 g_2^s - \frac{s}{2} \partial_z g_1^s - m_I g_0^s - \frac{1}{2} m_R' \partial_{k_z} g_3^s = 0. \tag{27}$$

We first use the constraint equations (25-27) iteratively up to first order to express g_1^s, g_2^s and g_3^s in terms of g_0^s and $\partial_{k_z} g_0^s$, and then insert the results into (24). Remarkably all terms proportional to $\partial_{k_z} g_0^s$ cancel and we find that to first order in gradients g_0^s satisfies the algebraic equation

$$\left(k_0^2 - k_z^2 - |m|^2 + \frac{s}{k_0}|m|^2\theta'\right) g_0^s = 0\,. \tag{28}$$

This admits the spectral solution $g_0^s = \pi n_s |k_0| \delta(\Omega_s^2)$, which can also be written as

$$g_0^s \;=\; \sum_{\pm} \frac{\pi}{2Z_{s\pm}}\, n_s\, \delta(k_0 \mp \omega_{s\pm})\,. \tag{29}$$

Here $n_s(k_0, k_z, z)$ are nonsingular functions which are, as we show below, related to the on-shell distribution functions. The indices $\pm$ refer to the sign of k_0, to be eventually related to particles and antiparticles. The energy $\omega_{s\pm}$ is specified by the roots of the equation

$$\Omega_s^2 \;\equiv\; k_0^2 - k_z^2 - |m|^2 + \frac{s}{k_0}|m|^2\theta' \;=\; 0, \tag{30}$$

and the normalization factor $Z_{s\pm}$ is defined as

$$Z_{s\pm} \equiv \frac{1}{2\omega_{s\pm}} |\partial_{k_0} \Omega_s^2|_{k_0 = \pm\omega_{s\pm}}. \tag{31}$$

To first order in gradients these can be solved iteratively:

$$\omega_{s\pm} = \omega_0 \mp s\frac{|m|^2\theta'}{2\omega_0^2}\,, \qquad \omega_0 = \sqrt{k_z^2+|m|^2} \tag{32}$$

$$Z_{s\pm} = 1 \mp s\frac{|m|^2\theta'}{2\omega_0^3} = \frac{\omega_{s\pm}}{\omega_0}\,. \tag{33}$$

Equation (32) defines the physical dispersion relations for particles and antiparticles of a given spin s. Due to the derivative corrections the spin degeneracy is lifted at first order in gradients and hence particles (antiparticles) of different spin experience different accelerations in a spatially varying background, as we shall see in more detail below.

Solution (29) nicely illustrates how the constraint equations operate. Solving (24-27) consistently to first order accuracy constrains the solutions of the kinetic equation to sharp, locally varying energy shells given by (32). One way of understanding this is as follows. The CP-violating phase θ' in (28) can be related to an axial gauge field [29, 15] which, to leading order in gradients, lifts the degeneracy in the dispersion relation, but does not spoil the quasiparticle picture, just as it is the case with a vector gauge field. We should note however, that the confinement to sharp energy shells does not persist beyond first order in gradients. While for noninteracting fermions one can always express $g^s_{1,2,3}$ in terms of g^s_0, at higher orders more complicated derivative structures arise, as the constraint cannot be written as a simple algebraic equation with a spectral solution. For a treatment of such a situation in the case of a scalar field see [30].

Eq. (32) is identical with the results derived earlier by WKB-methods in [22] and simultaneously via the field-theoretic technique of spectral integrals in [28]. From the WKB-point of view the present derivation comes as a welcome verification of the result obtained by an intuitive, but less fundamental approach. The agreement with the field theoretical calculation of [28] on the other hand was to be expected. In [28] it was shown that the integral over the spectral function, defined as a difference of the retarded and advanced Green functions $\mathcal{A} = (i/2)(G_{\rm ret} - G_{\rm adv})$, projects test functions onto energy shells (32); however, in the collisionless limit $\mathcal{A}$ satisfies the same equation of motion (8) as the Wightman function $G^<$. The trace of $\gamma^0\mathcal{A}$ in particular satisfies equation (28), and can be obtained from (29) by the replacement $n_s \to 1$. We can hence immediately check the sum-rule to the accuracy

at which we are working. Indeed,

$$\int_{-\infty}^{\infty} \frac{dk_0}{\pi} \mathrm{Tr}\, \gamma^0 \mathcal{A}_s \;=\; \sum_{\pm} \int_{-\infty}^{\infty} \frac{dk_0}{2Z_{s\pm}} \delta(k_0 \mp \omega_{s\pm}) \;=\; 1. \tag{34}$$

We finally note that equation (30) has additional poles at $k_0 \simeq s|m|^2\theta'/2\omega_0$, which we have left out in the decomposition (29). These poles correspond to unphysical, but harmless, tachyonic modes which arise only because our solutions for the constraint equations involve an expansion in inverse powers of k_0, which breaks down already for k_0 much larger than the value associated with these poles. Note that their contribution to the sum rule (34) vanishes when summed over spins.

2.2 Kinetic equations

We now turn our attention to the kinetic equations. We are primarily interested in the equation for g_0^s which carries information on the particle density in phase space. From (19) we have

$$\partial_t g_0^s + s\partial_z g_3^s - m_R' \partial_{k_z} g_1^s - m_I' \partial_{k_z} g_2^s = 0, \tag{35}$$

which is correct up to second order in gradients (first order in $\hbar$). Just as in the previous section we use the constraint equations (25-27) to express g_1^s, g_2^s and g_3^s in terms of g_0^s and arrive at an equation for g_0^s alone. To second order in gradients (first order in $\hbar$) it reads

$$k_0 \partial_t g_0^s + k_z \partial_z g_0^s - \left(\frac{1}{2} |m|^{2\,\prime} - \frac{s}{2k_0} (|m|^2 \theta')' \right) \partial_{k_z} g_0^s = 0. \tag{36}$$

We have so far used three out of four constraint equations. To find the acceptable solutions satisfying all constraints, we must yet impose the restriction onto the functional space spanned by (29). Because of the δ-function in the decomposition (29) this is of course trivial. All we need to do is to insert (29) into (36) and integrate over the positive and negative frequencies k_0. We then get the following form for the Liouville equation

$$\partial_t f_{s\pm} + v_{s\pm} \partial_z f_{s\pm} + F_{s\pm} \partial_{k_z} f_{s\pm} = 0, \tag{37}$$

where

$$\begin{aligned} f_{s+} &\equiv n_s(\omega_{s+}, k_z, z) \\ f_{s-} &\equiv 1 - n_s(-\omega_{s-}, -k_z, z) \end{aligned} \tag{38}$$

are the distribution functions for particles and antiparticles with spin s, respectively. These definitions are motivated by the equilibrium result $n_s^{\rm eq} = 1/(e^{\beta k_0}+1)$, where $\beta = 1/T$ is the inverse temperature. The quasiparticle *group velocity* $v_{s\pm}$ appearing in Eq. (37) is given by

$$v_{s\pm} = \frac{k_z}{\omega_{s\pm}}, \tag{39}$$

where k_z is the kinetic momentum. The spin-dependent and CP-violating *semiclassical force* reads

$$F_{s\pm} = -\frac{|m|^{2\,\prime}}{2\omega_{s\pm}} \pm \frac{s(|m|^2\theta')'}{2\omega_0^2}. \tag{40}$$

Equations (37-40) are among the main results of this paper. Incidentally, the form (40) for the semiclassical force $F_{s\pm} = \omega_{s\pm} dv_{s\pm}/dt$ was already found by Joyce, Prokopec and Turok [29]. To obtain kinetic equations by WKB-methods the authors of [15] used canonical variables however. The resulting equations are not invariant under reparametrization of the wave functions, and hence care is required when specifying local thermal equilibrium in derivation of transport equations relevant for baryogenesis. Cline, Joyce and Kainulainen [22] introduced the kinetic momentum as a physical variable in the kinetic equations and obtained the unique reparametrization invariant transport equations identical with (37) and (39-40). The outstanding contribution of the present work is in a controlled first principle derivation of these equations without any *a priori* assumptions.

Let us finally note that equation (36) could have been obtained by taking the bilinear $\diamond$-derivative of the constraint equation (28), where the $\diamond$-derivative is defined by

$$\diamond\{a\}\{b\} \equiv \frac{1}{2}\left(\partial_t a\,\partial_{k_0} b - \partial_z a\,\partial_{k_z} b - \partial_{k_0} a\,\partial_t b + \partial_{k_z} a\,\partial_z b\right). \tag{41}$$

This is no coincidence, and even more generally, in the collisionless limit the kinetic equation can be obtained by effecting the tan $\diamond$-derivative on the constraint equation.

2.3 Currents

It is instructive to study the expressions for physical currents in order to shed light on various functions we have encountered in our derivation. Of particular relevance for baryogenesis are the vector and axial vector currents. By making use of (6) and (15-16) one finds

$$j^\mu \;\; \equiv \;\; \left\langle \bar\psi(x)\gamma^\mu\psi(x)\right\rangle = \sum_{s=\pm1}\int \frac{d^2k}{(2\pi)^2}\,(\,g_0^s, s g_3^s\,)$$

$$j_5^\mu \equiv \left\langle \bar{\psi}(x)\gamma^\mu\gamma^5\psi(x)\right\rangle = \sum_{s=\pm 1}\int \frac{d^2k}{(2\pi)^2}\,(\,g_3^s, sg_0^s\,)\,, \tag{42}$$

where we have restricted ourselves to 1+1-dimensions so that $d^2k = dk_z dk_0$. This shows that g_0^s is the usual number density in phase space, whereas g_3^s represents the axial charge density. An important consequence of the constraint equations is that there is only one independent dynamical function, here chosen to be g_0^s, while all others can be related to g_0^s via the constraint equations (25-27). In particular, g_3^s can be written as

$$g_3^s = \Big(s\frac{k_z}{k_0} + \frac{1}{2k_0^2}|m|^2\theta'\partial_{k_z}\Big)\, g_0^s\,. \tag{43}$$

The nontrivial gradient correction appears as a total derivative and hence vanishes upon the k_z-integration. Using the decomposition (29) one finds

$$j_{s\pm}^\mu = \int \frac{dk_z}{8\pi Z_{s\pm}}\,(1, v_{s\pm})\, f_{s\pm} = \sum_{s_{k_z}=\pm} s_{k_z}\int\frac{d\omega}{8\pi}\,(\frac{1}{v_{s\pm}},1)\, f_{s\pm} \tag{44}$$

$$j_{5s\pm}^\mu = s\int \frac{dk_z}{8\pi Z_{s\pm}}\,(v_{s\pm},1)\, f_{s\pm} = s\sum_{s_{k_z}=\pm} s_{k_z}\int\frac{d\omega}{8\pi}\,(1,\frac{1}{v_{s\pm}})\, f_{s\pm}\,, \tag{45}$$

where s_{k_z} denotes the sign of k_z, and we discarded the vacuum contribution. In the last step we used $\partial_{k_z}\omega_{s\pm} = Z_{s\pm}^{-1}k_z/\omega_{s\pm}$. The lower limit in the ω-integrals is $|m| \mp s\theta'/2$. The functions $f_{s\pm}$ are the correctly normalized distribution functions, and they retain the correct physical interpretation in that the particle flux is not affected by CP-violating effects, while the density may be either enhanced or suppressed, as given by the inverse velocity. The current (44) was computed by WKB-methods in [22]. The correct result for $j_{s\pm}^0$ was also found in [28] by field-theoretical methods. However, $j_{s\pm}^3$ found in [28] does not agree with (44) because the spinor structure used for $i\gamma^0 G^<$ was too simple.

2.4 Interactions

In all discussions above, we have left out the effects of interactions. This was done to avoid the necessity to use the full machinery of the Schwinger-Keldysh formalism [38], and to keep things as simple as possible. Moreover, unlike the treatment of the flow term of the kinetic equation presented above, including interactions is necessarily a model dependent

task. Nevertheless it is a simple matter to write a *formally exact* equation of motion for $G^<$ including the collision terms. Instead of (8) we then have

$$(i\,\partial\!\!\!/ - m_R - im_I\gamma^5)\,iG^< - \Sigma_{\rm R}\odot iG^< \;=\; \Sigma^<\odot G_R + \frac{1}{2}\Big(\Sigma^>\odot G^< - G^<\odot\Sigma^>\Big), \qquad (46)$$

where $A\odot B(u,v) \;\equiv\; \int dw\;\; A(u,w)B(w,v)$. G_R is the real part of the (retarded) propagator, and the function $\Sigma_{\rm R}$ contains the real part of the self-energy corrections including the singular (tadpole) interactions which can be resummed to a renormalized mass term. The terms in parentheses give rise to the usual collision term, where the self-energies $\Sigma^{<,>}$ arise only from nonlocal loop contributions to the Dyson-Schwinger equations. However, as mentioned above, the exact form of these terms depends on the theory considered. While their treatment is not conceptually difficult, their inclusion brings a considerable amount of technical complications. We shall consider the problem of including collisions elsewhere [27].

3 Mixing fermionic fields

In practical applications, such as in supersymmetric models, one needs to consider cases where several fermion flavours are mixed by a spatially varying mass matrix. We therefore consider a theory with the mass lagrangian

$$\mathcal{L}_{\rm mass} \;=\; -\bar\psi_L M\psi_R - \bar\psi_R M^\dagger\psi_L\,, \qquad (47)$$

where M is a complex (in general nonhermitean) $N\times N$ matrix with spatially varying components. We denote the flavour degree of freedom by an additional index i to the spinor $\psi_{\alpha,i}(x)$, so the Wightman function becomes a matrix in the product space of spinor and flavour:

$$G^<_{\alpha\beta,ij}(u,v) = i\left\langle\bar\psi_{\beta,j}(v)\psi_{\alpha,i}(u)\right\rangle\;. \qquad (48)$$

The flavour degree of freedom plays no role in the derivation of the equations of motion for $G^<_{\alpha\beta,ij}$ in the steps analogous to going from (8) to (19-22) in the single fermion field case, because those steps dealt only with the spinor structure of $G^<$. We can thus immediately write an equation analogous to (18):

$$\left(\hat k_0 - s\hat k_z\rho^3 - \rho^1\hat M_0 - \rho^2\hat M_5\right)g_s^< = 0. \qquad (49)$$

The sole, but significant difference to (18) is that $g_s^<$ is now an $N \times N$-matrix in the flavour space, and the mass terms have become $N \times N$-matrix operators

$$\hat{M}_0 = \frac{1}{2}(\hat{M} + \hat{M}^\dagger) \tag{50}$$

$$\hat{M}_5 = -\frac{i}{2}(\hat{M} - \hat{M}^\dagger) \tag{51}$$

where $\hat{M} \equiv M e^{\frac{i}{2}\overleftarrow{\partial}_z \partial_{k_z}}$ and $\hat{M}^\dagger \equiv M^\dagger e^{\frac{i}{2}\overleftarrow{\partial}_z \partial_{k_z}}$. The extra flavour structure of course complicates the solution of equation (49) and it turns out to be convenient to perform a rotation to the basis where the lowest order mass matrix is diagonal. Because M in general can be nonhermitean, the diagonalization requires a *biunitary* transformation

$$U M V^\dagger = M_d\,, \tag{52}$$

where U and V are the unitary matrices which diagonalize the hermitean matrices $MM^\dagger$ and $M^\dagger M$, respectively. After the rotation we can write (49) in the component form in the diagonal basis as

$$(k_0 + \frac{i}{2}\mathcal{D}_t^-)g_{0d}^s - \ s(k_z - \frac{i}{2}\mathcal{D}_z^-)g_{3d}^s - \ \hat{M}_{0d}g_{1d}^s - \ \hat{M}_{5d}g_{2d}^s = 0 \tag{53}$$

$$(k_0 + \frac{i}{2}\mathcal{D}_t^-)g_{3d}^s - \ s(k_z - \frac{i}{2}\mathcal{D}_z^-)g_{0d}^s - i\hat{M}_{0d}g_{2d}^s + i\hat{M}_{5d}g_{1d}^s = 0 \tag{54}$$

$$(k_0 + \frac{i}{2}\mathcal{D}_t^+)g_{1d}^s + is(k_z - \frac{i}{2}\mathcal{D}_z^+)g_{2d}^s - \ \hat{M}_{0d}g_{0d}^s - i\hat{M}_{5d}g_{3d}^s = 0 \tag{55}$$

$$(k_0 + \frac{i}{2}\mathcal{D}_t^+)g_{2d}^s - is(k_z - \frac{i}{2}\mathcal{D}_z^+)g_{1d}^s + i\hat{M}_{0d}g_{3d}^s - \ \hat{M}_{5d}g_{0d}^s = 0. \tag{56}$$

The ‘covariant derivatives’ appearing in (53-56) are defined as

$$\mathcal{D}_t^\pm \equiv \partial_t - i[\Sigma_t,\,\cdot\,]_- - is[\Delta_z,\,\cdot\,]_\pm \tag{57}$$

$$\mathcal{D}_z^\pm \equiv \partial_z - i[\Sigma_z,\,\cdot\,]_- - is[\Delta_t,\,\cdot\,]_\pm \tag{58}$$

where the brackets $[\cdot,\cdot]_-$ refer to commutators and $[\cdot,\cdot]_+$ to anticommutators and

$$\Sigma_\mu \equiv \frac{i}{2}(V\partial_\mu V^\dagger + U\partial_\mu U^\dagger) \tag{59}$$

$$\Delta_\mu \equiv \frac{i}{2}(V\partial_\mu V^\dagger - U\partial_\mu U^\dagger)\,. \tag{60}$$

It should be noted that, while the relation (52) allows arbitrary phase redefinitions $U \to wU$ and $V \to wV$, where w is any diagonal matrix with $|w_{ii}| = 1$, the operator Δ_μ remains invariant under these transformations. This reparametrization freedom is exactly what leads to the apparent 'gauge' dependence of the results in the WKB-approach [22]. As we show below, only the diagonal elements of Δ_μ contribute to the constraint and kinetic equations to order $\hbar$, which then implies that our results are reparametrization invariant. Finally, the new mass operators in the diagonal basis are given by

$$\hat{M}_{0d} = \frac{1}{2}\left(U\hat{M}V^\dagger + V\hat{M}^\dagger U^\dagger\right) \tag{61}$$

$$\hat{M}_{5d} = -\frac{i}{2}\left(U\hat{M}V^\dagger - V\hat{M}^\dagger U^\dagger\right). \tag{62}$$

The constraint and kinetic equations now correspond to the *hermitean* and *antihermitean* parts of (53-56), respectively. Because of the matrix structure, these equations contain a number of commutator and anticommutator terms involving g_a^s and the various matrix-operators. However, we shall now argue that (53-56) can effectively be taken to be diagonal to the order at which we are working. Indeed, in the propagating basis (52) the off-diagonal terms are obviously suppressed by $\hbar$ when compared to the diagonal elements. On the other hand, they appear in the diagonal equations through the commutators $\hbar^{-1}[\hbar\Sigma_z, g_{ad}^s] \equiv [\Sigma_z, g_{ad}^s]$ and $[\Delta_z, g_{ad}^s]$, and thus at the same order as the diagonal elements. This then immediately implies that, when the dynamics of CP-violating densities is considered, the off-diagonals contribute at second order in $\hbar$ in the diagonal equations and can be consistently neglected. With this it is now straightforward to show that, to first order accuracy in $\hbar$, the constraint equations reduce to the following equations for the *diagonal* entries of g_{ad}^s:

$$k_0 g_{0d}^s - s k_z g_{3d}^s - m_R g_{1d}^s - m_I g_{2d}^s = 0 \tag{63}$$

$$k_0 g_{3d}^s - s k_z g_{0d}^s + \frac{1}{2}\tilde{m}_R' \partial_{k_z} g_{2d}^s - \frac{1}{2}\tilde{m}_I' \partial_{k_z} g_{1d}^s = 0 \tag{64}$$

$$k_0 g_{1d}^s + \frac{s}{2}\partial_z g_{2d}^s + s\Delta_{zd} g_{1d}^s - m_R g_{0d}^s + \frac{1}{2}\tilde{m}_I' \partial_{k_z} g_{3d}^s = 0 \tag{65}$$

$$k_0 g_{2d}^s - \frac{s}{2}\partial_z g_{1d}^s + s\Delta_{zd} g_{2d}^s - m_I g_{0d}^s - \frac{1}{2}\tilde{m}_R' \partial_{k_z} g_{3d}^s = 0, \tag{66}$$

where $m_{R,I}$ are the real and imaginary parts of the eigenvalues of M_d, Δ_{zd} is the diagonal part of Δ_z and $\tilde{m}_R' \equiv m_R' - 2m_I\Delta_{zd}$ and $\tilde{m}_I' \equiv m_I' + 2m_R\Delta_{zd}$. Similarly, the kinetic equation

for g^s_{0d} to first order in $\hbar$ becomes

$$\partial_t g^s_{0d} + s\partial_z g^s_{3d} - \tilde{m}'_R \partial_{k_z} g^s_{1d} - \tilde{m}'_I \partial_{k_z} g^s_{2d} = 0. \tag{67}$$

Following our treatment in section 2, it is now straightforward to eliminate g^s_{ad} $(a = 1, 2, 3)$ from equations (63) and (67) to obtain the constraint equation

$$\Big(k_0^2 - k_z^2 - |M_d|^2 + \frac{s}{k_0}|M_d|^2\Theta'\Big)\, g^s_{0d} = 0 \tag{68}$$

and the kinetic equation

$$k_0\partial_t g^s_{0d} + k_z\partial_z g^s_{0d} - \Big(\frac{1}{2}|M_d|^{2\prime} - \frac{s}{2k_0}(|M_d|^2\Theta')'\Big)\partial_{k_z} g^s_{0d} = 0 \tag{69}$$

for the number density function g^s_{0d} alone. Eqs. (68) and (69) are analogous to the one field equations (28) and (36). The difference is that here we have N different equations corresponding to N diagonal elements of g^s_{0d} in the mass eigenbasis. The derivative of the effective angle Θ' appearing in (68-69) is defined as

$$\Theta' = \Theta'_d + 2\Delta_{zd} \tag{70}$$

where the angles Θ_d are the complex phases of the elements in the diagonal mass matrix: $M_d \equiv |M_d|e^{i\Theta_d}$. Since the mixing field contribution shows up as a shift in the derivative of the pseudoscalar phase $2\Delta_{zd}$, it implies that equations (68-69) are manifestly reparametrization invariant. We can convert (70) to an alternative form in terms of the original mass matrix and the rotation matrix U:

$$|M_d|^2\Theta' = -\frac{1}{2}\mathrm{Im}\Big(U(MM'^{\dagger} - M'M^{\dagger})U^{\dagger}\Big)_d\,, \tag{71}$$

which is also manifestly reparametrization invariant. This expression will be convenient for discussion of the chargino sector of the MSSM and the NMSSM in sections 3.1 and 3.2.

The final steps in going from equations (68) and (69) to kinetic equations for the mass eigenmodes are exactly analogous to the single fermionic field case: each mass eigenmode i gets projected to its own energy shell $\omega_{si\pm}$ given by (68), and the corresponding spectral decomposition density function $f_{si\pm}$ obeys a semiclassical Boltzmann equation identical to (37)

$$\partial_t f_{si\pm} + v_{si\pm}\partial_z f_{si\pm} + F_{si\pm}\partial_{k_z} f_{si\pm} = 0\,, \tag{72}$$

with the corresponding group velocity

$$v_{si\pm} = \frac{k_z}{\omega_{si\pm}} \tag{73}$$

and the CP-violating semiclassical force

$$F_{si\pm} = -\frac{|M_i|^{2\prime}}{2\omega_{si\pm}} \pm \frac{s(|M_i|^2\Theta_i')'}{2{\omega_{0i}}^2} \tag{74}$$

computed from expression (70) or (71). It has been shown in [29, 15, 22], that the spin-dependent term in $F_{si\pm}$ gives rise to a CP-violating source proportional to $|M_i|^2\Theta_i'$ in the diffusion equations. We can therefore loosely call this factor the 'source', and proceed to compute it in some special cases.

3.1 Charginos in the MSSM

We first compute the source in the transport equations for charginos in the MSSM. The chargino mass term reads

$$\overline{\Psi}_R\, M\, \Psi_L + \text{h.c.}\,, \tag{75}$$

where $\Psi_R = (\tilde{W}_R^+, \tilde{h}_{1,R}^+)^T$ and $\Psi_L = (\tilde{W}_L^+, \tilde{h}_{2,L}^+)^T$ are the chiral fields in the basis of winos. The mass matrix reads

$$M = \begin{pmatrix} m_2 & gH_2^* \\ gH_1^* & \mu \end{pmatrix}, \tag{76}$$

where H_1 and H_2 are the Higgs field vacuum expectation values and μ and m_2 are the soft supersymmetry breaking parameters. For a realistic choice of parameters there is no spontaneous CP-violation in the MSSM, so to a good approximation we can take the Higgs *vev's* to be real [25, 23]. The matrix U in (52) can be parametrized as [22]

$$U = \frac{\sqrt{2}}{\sqrt{\Lambda(\Lambda+\Delta)}} \begin{pmatrix} \frac{1}{2}(\Lambda+\Delta) & a \\ -a^* & \frac{1}{2}(\Lambda+\Delta) \end{pmatrix} \tag{77}$$

with

$$\begin{aligned} a &= g(m_2 H_1 + \mu^* H_2^*) && (78)\\ \Delta &= |m_2|^2 - |\mu|^2 + g^2(h_2^2 - h_1^2) && (79)\\ \Lambda &= \sqrt{\Delta^2 + 4|a|^2}\,, && (80) \end{aligned}$$

where $h_i \equiv |H_i|$ are normalized such that the tree level W-boson mass is $M_W^2 = g^2 h^2/2$, $h^2 = h_1^2 + h_2^2$. The physical chargino mass eigenvalues are given by

$$m_\pm^2 = \frac{1}{2}\left(|m_2|^2 + |\mu|^2 + g^2 h^2\right) \pm \frac{\Lambda}{2}\,. \tag{81}$$

Upon inserting (76) and (77) into (71) it is straightforward to show that the source term for charginos becomes

$$m_\pm^2 \Theta'_\pm = \mp\frac{g^2}{\Lambda}\Im(\mu m_2)(h_1 h_2)'\,, \tag{82}$$

where Θ_+ (Θ_-) corresponds to the higgsino-like state when $|\mu| > |m_2|$ ($|\mu| < |m_2|$). CP violation is here mediated via the parameters μ, m_2 and may in fact be large [39]. The result (82) is in perfect agreement with the chargino source obtained by a WKB method in Ref. [22].

3.2 Charginos in the NMSSM

In the NMSSM there is an additional singlet field S in the Higgs sector. The singlet field couples to higgsinos, and hence the higgsino-higgsino component in the chargino mass matrix (76) is generalized in the NMSSM:

$$\mu \to \tilde{\mu} \equiv \mu + \lambda S, \tag{83}$$

where λ is the coupling for higgs(ino)-higgs(ino)-singlet interaction. Another consequence of this extension is the possibility to have spontaneous transitional CP-violation [25], so the Higgs fields H_i are in general complex. When the parameters a, Δ and Λ are defined as in equation (80) with $\mu \to \tilde{\mu}$, the matrix U in equation (77) still diagonalizes $MM^\dagger$.

In the NMSSM we must account for the complex dynamical phases of the higgs fields, and hence we have to be more careful with our definition of the higgs doublets. Our choice of writing the mass matrices (76) corresponds to parametrizing the higgs doublets Φ_i as [40]:

$$\Phi_1 = \begin{pmatrix} h_1 e^{i\theta_1} \\ h_1^- \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} h_2^+ \\ h_2 e^{i\theta_2} \end{pmatrix}, \tag{84}$$

where $h_i^\pm$ are the charged higgs fields. Only one of the higgs phases θ_i is physical, while the other gets eaten by the gauge fields in the unitary gauge. We wish to choose the physical

phase in such a way that the corresponding field does not couple to the neutral weak boson. Given the parametrization (84), this condition implies that

$$h_1^2\theta_1' = h_2^2\theta_2'. \tag{85}$$

Using the gauge constraint (85) we can write

$$\theta_1' = \frac{h_2^2}{h^2}\theta' \,, \qquad \theta_2' = \frac{h_1^2}{h^2}\theta' \,, \tag{86}$$

where $h^2 = h_1^2 + h_2^2$, and $\theta = \theta_1 + \theta_2$ is the physical CP-violating phase.

To get the explicit form for the CP-violating term, we insert the NMSSM mass matrix (76) into (71). After some algebra one finds the following three terms giving rise to CP-violating sources in the NMSSM:

$$\Theta'_{\rm NMSSM} = \Theta'_{h_1h_2} + \Theta'_\theta + \Theta'_S \,, \tag{87}$$

The first term is the following generalization of the chargino source (82):

$$m_\pm^2\Theta'_{h_1h_2\pm} = \mp\frac{g^2}{\Lambda}\Im(\tilde\mu m_2 e^{i\theta})\,(h_1h_2)' \tag{88}$$

for the case involving a new scalar field S and possibly complex higgs fields. However, there are two new types of terms in the NMSSM. The term Θ'_θ is proportional to a derivative of the CP-violating phase θ in the Higgs sector, and reads

$$m_\pm^2\Theta'_{\theta\pm} = -\frac{g^2\theta'}{\Lambda}\left(\Big(\Lambda \pm (|m_2|^2 + |\tilde\mu|^2)\Big)\frac{h_1^2h_2^2}{h^2} \mp \Re(\tilde\mu m_2 e^{i\theta})h_1h_2\right). \tag{89}$$

Finally, the source Θ'_S can be written as a derivative of the singlet condensate:

$$m_\pm^2\Theta'_{S\pm} = \pm\frac{\lambda g^2}{\Lambda}\Im(m_2H_1H_2S') + \frac{\lambda g^2}{2\Lambda}\Big(\Lambda \pm (|\tilde\mu|^2 + g^2h^2 - |m_2|^2)\Big)\Im(\tilde\mu^* S')\,. \tag{90}$$

In all formulae (88-90) the mass eigenvalues $m_\pm^2$ can be read off from equation (81) with the replacement $\mu \to \tilde\mu$, where $\tilde\mu$ is given by Eq. (83). Baryogenesis in the NMSSM from the semiclassical force has been studied in Ref. [24].

4 Mixing bosonic fields

Here we first show that, unlike for fermions, the constraint and kinetic equations for mixing bosons acquire no gradient correction to first order in $\hbar$ in the collisionless limit. We then derive the constraint and kinetic equations accurate to second order in gradients which can be used as a starting point for baryogenesis calculations. N mixing bosonic fields with a spatially varying mass matrix M^2 obey the Klein-Gordon equation

$$(\Box_u + M^2(u))\phi(u) = 0, \tag{91}$$

where ϕ is an N-dimensional vector whose components are coupled by the hermitean mass matrix M^2. Multiplying (91) from the left by $-i\phi^\dagger(v)$ and taking the expectation value with respect to the initial state we get

$$(\Box_u + M^2(u))G^<(u,v) = 0. \tag{92}$$

where the Wightman function $G^<$ is defined as

$$G^<(u,v) = -i\langle\phi^\dagger(v)\phi(u)\rangle. \tag{93}$$

After performing the Wigner transform, equation (92) becomes

$$\left(\frac{1}{4}\partial^2 - k^2 - ik\cdot\partial + M^2 e^{-\frac{i}{2}\overleftarrow{\partial}\cdot\partial_k}\right)G^< = 0. \tag{94}$$

In the case when $N = 1$ it is immediately clear that the first quantum correction to the constraint equation (the real part of (94)) is of second order and to the kinetic equation (imaginary part) of third order in gradients (second order in $\hbar$). To extract the spectral information to second order in $\hbar$ is quite delicate since the constraint equation in (94) contains derivatives [30]. In the case of more than one mixing fields it is convenient to rotate into the mass eigenbasis, just as in the fermionic case:

$$M_d^2 = UM^2U^\dagger, \tag{95}$$

where U is a unitary matrix. In the propagating basis the equation (94) becomes

$$\left(\frac{1}{4}\mathcal{D}^2 - k^2 - ik\cdot\mathcal{D} + M^2 e^{-\frac{i}{2}\overleftarrow{\mathcal{D}}\cdot\partial_k}\right)G_d^< = 0, \tag{96}$$

where $G_d^< \equiv U G^< U^\dagger$ and the 'covariant' derivative is defined as:

$$\mathcal{D}_\mu = \partial_\mu - i\,[\Xi_\mu,\ \cdot\ ]\,, \qquad \Xi_\mu = iU\partial_\mu U^\dagger. \tag{97}$$

Since $(G_d^<)^\dagger = -G_d^<$ and $\mathcal{D}_\mu^\dagger = \mathcal{D}_\mu$, we identify the antihermitean part of (96) as the constraint equation:

$$-2k^2 G_d^< + \left\{\hat{M}_c^2 + \frac{1}{4}\mathcal{D}^2, G_d^<\right\} - i\left[k\cdot\mathcal{D} + \hat{M}_s^2, G_d^<\right] = 0, \tag{98}$$

and the hermitean part is the kinetic equation

$$\left\{k\cdot\mathcal{D} + \hat{M}_s^2, G_d^<\right\} - i\left[\hat{M}_c^2 + \frac{1}{4}\mathcal{D}^2, G_d^<\right] = 0, \tag{99}$$

where we defined

$$\begin{aligned} \hat{M}_c^2 &= M_d^2 \cos\frac{1}{2}\overleftarrow{\mathcal{D}}\cdot\partial_k \\ \hat{M}_s^2 &= M_d^2 \sin\frac{1}{2}\overleftarrow{\mathcal{D}}\cdot\partial_k. \end{aligned} \tag{100}$$

We now use the analogous argument as in the fermionic case in section 3. The off-diagonal elements of $G_d^<$ in (98-99) are sourced by the diagonal elements through the terms involving commutators which are suppressed by at least $\hbar$ with respect to the diagonal elements. This implies that, in order to capture the leading order nontrivial effect in gradients, we can work in the diagonal (semiclassical) approximation for $G_d^<$. By inspection of (98-99) we can now immediately write the constraint and kinetic equations in the diagonal approximation accurate to order $\hbar$ as follows

$$\left(k^2 - M_d^2\right) G_d^< = 0 \tag{101}$$

$$\left(k\cdot\partial + \frac{1}{2}(\partial M_d^2)\cdot\partial_k\right) G_d^< = 0. \tag{102}$$

In contrast to the fermionic equivalent (68), these equations contain no CP-violating corrections to order $\hbar$ and display only the usual classical CP-conserving term associated with the mass eigenvalues. This analysis is relevant for example for calculation of the CP-violating force in the *stop* sector $\tilde{q} = (\tilde{t}_L, \tilde{t}_R)^T$ of the MSSM, in which the mass matrix reads

$$M_{\tilde{q}}^2 = \begin{pmatrix} m_Q^2 & y(A^* H_2 + \mu H_1) \\ y(AH_2 + \mu^* H_1) & m_U^2 \end{pmatrix}, \tag{103}$$

where m_Q^2 and m_U^2 denote the sum of the soft susy-breaking masses, including D-terms and $m_t^2 = y^2 H_2^2$. Our analysis immediately implies that for squarks there is no CP-violating correction to the dispersion relation at first order in gradients, and hence there is no CP-violating semiclassical force in the kinetic equation at order $\hbar$.

5 Continuity equations and CP-violating sources

The quantities eventually relevant for baryogenesis are CP-violating fluxes. We now investigate how the CP-violating sources appear in the equations for the divergences of the vector and axial vector currents. Let us first consider the divergence of the vector current. We have

$$\begin{aligned} \partial_\mu j^\mu &= \partial_\mu \langle \bar{\psi}(x) \gamma^\mu \psi(x) \rangle && (104) \\ &= \int \frac{d^2k}{(2\pi)^2} \partial_\mu \mathrm{Tr}(-iG^< \gamma^\mu) && (105) \end{aligned}$$

where the derivative in the integral expression is taken with respect to the center-of-mass coordinate. Using the decomposition (16) and the constraint equations to write g_{3i}^s in terms of g_{0i}^s (i is the flavour index), Eq. (16) is easily shown to reduce to just a momentum integral over the kinetic equation (69). Hence we get the usual continuity equation

$$\partial_\mu j_{si\pm}^\mu \equiv \partial_t n_{si\pm} + \partial_z (n_{si\pm} u_{si\pm}) = 0, \tag{106}$$

showing that the vector current is conserved, and contains no sources. The fluid density $n_{si\pm}$ and velocity $u_{si\pm} \equiv \langle v_{si\pm} \rangle$ are defined as

$$\begin{aligned} n_{si+} &\equiv \int_+ \frac{d^2k}{(2\pi)^2} g_{0i}^s \\ n_{si+} \langle v_{si+}^p \rangle &\equiv \int_+ \frac{d^2k}{(2\pi)^2} \left(\frac{k_z}{k_0} \right)^p g_{0i}^s \,, && (107) \end{aligned}$$

where $\int_+ \equiv \int_{k_0 \geq 0}$ denotes integration over the positive frequencies. The fluid density $n_{si\pm}$ should not be confused with the phase space density n_s in (38). To get the density and velocity moments for antiparticles one should integrate over the negative frequencies and make use of (38). Here we have again restricted ourselves to 1+1 dimensions; in 3+1 dimensions the expressions differ in detail, but not in essence [27]. No source appears in (106)

simply because the semiclassical force term in (69) reduces to vanishing boundary terms at $k_z \to \pm\infty$. In the case of a single Dirac fermion this result remains valid to any order in gradients. In a general case of mixing fields the dynamics of off-diagonal elements of g_0^s may induce sources, which would however be higher than first order in $\hbar$. Our proof that there are no CP-violating sources to the continuity equation for the vector current is contrary to the results of Refs. [37, 17]. Our derivation is more general than that of [37] in that by treating mass as part of the flow term it includes the "mass resummation" of Refs. [37, 17] to infinite order, it is not based on any particular *Ansatz* for g_{0d}^s, and finally, but most importantly, we took correct account of the constraint equations. The fact that we have not treated collisions terms here does not resolve the differences, because the collisional contributions arise only from nonsingular loop diagrams which were not treated in Ref. [37] either.

We next consider the continuity equation for the axial vector current. This can be obtained from Eq. (106) by replacing γ^μ by $\gamma^\mu\gamma^5$. The presence of γ^5 essentially changes the roles of g_3^s and g_0^s, so that the axial divergence reduces to an integral over the kinetic equation for g_3^s, which can be inferred from (54):

$$\partial_t g_3^s + s\partial_z g_0^s + 2\left(M_I - \frac{1}{8}M_I''\partial_{k_z}^2\right) g_1^s - 2\left(M_R - \frac{1}{8}M_R''\partial_{k_z}^2\right) g_2^s = 0. \tag{108}$$

It is thus easy to see that the divergence of the axial current acquires the expected form:

$$\partial_\mu j_{5s}^\mu = -2iM_R\langle\bar{\psi}_s\gamma^5\psi_s\rangle - 2M_I\langle\bar{\psi}_s\psi_s\rangle, \tag{109}$$

where $\langle\bar{\psi}_s\psi_s\rangle = \int_k g_1^s$ denotes the scalar, and $\langle\bar{\psi}_s\gamma^5\psi_s\rangle = i\int_k g_2^s$ the pseudoscalar density, and $\int_k \equiv \int d^2k/(2\pi)^2$. We now make use of the constraint equations (53-56) to express g_1^s, g_2^s and g_3^s in (108) in terms of g_0^s (to second order in gradients), diagonalize (108) by rotating into the propagating basis, where we can take g_{0d}^s to be diagonal to order $\hbar$ accuracy, and finally integrate over the momenta. We thus arrive at the following equation for the axial current divergence:

$$\begin{aligned}\partial_t\left(n_{si\pm}u_{si\pm}\right) + \partial_z n_{si\pm} \;&-\; \left(|M_i|^2\partial_z + \frac{1}{2}|M_i|^{2\prime}\right)\mathcal{I}_{2si\pm}\\ &\pm\; s\left(|M_i|^2\Theta_i'\partial_z + \frac{1}{2}(|M_i|^2\Theta_i')'\right)\mathcal{I}_{3si\pm} = 0,\end{aligned} \tag{110}$$

where we defined

$$\mathcal{I}_{psi+} = \int_+ \frac{d^2k}{(2\pi)^2} \frac{g^s_{0i}}{k_0^p} \;=\; \int \frac{dk_z}{8\pi Z_{si\pm}} \frac{f_{si\pm}}{\omega^p_{si\pm}} \quad (p = 2, 3), \tag{111}$$

As usual, one has to introduce some truncation scheme to close the equations to two unknown quantities (here $n_{si\pm}$ and $u_{si\pm}$). There is of course some freedom as to how to do this step, and one might implement the truncation in (110) by replacing the distributions $f_{si\pm}$ in the $\mathcal{I}_{psi\pm}$-integrals by the equilibrium ones. However, it is instructive to observe that, by extracting a total derivative from the $|M_i|^2$-terms in (110), the corresponding integrals can be combined with the $\partial_z n_{si\pm}$-term to give a second velocity moment term, in terms of which (110) becomes simply

$$\partial_t \left(n_{si\pm} u_{si\pm}\right) + \partial_z \left(n_{si\pm} \langle v^2_{si\pm}\rangle\right) = S_{si\pm}, \tag{112}$$

where the source $S_{si\pm}$ is given by the average over the semiclassical force (74) divided by k_0:

$$S_{si\pm} = -\frac{1}{2}|M_i|^{2\prime}\mathcal{I}_{2si\pm} \pm \frac{1}{2}s(|M_i|^2\Theta_i')'\mathcal{I}_{3si\pm}. \tag{113}$$

This equation can be truncated by the method standardly used for moment expansion. One writes $\langle v^2_{si\pm}\rangle \to u^2_{si\pm} + \sigma^2_{si\pm}$ and uses the equilibrium distributions for $f_{si\pm}$'s when evaluating the variance $\sigma^2_{si\pm} \equiv \langle v^2_{si\pm}\rangle - u^2_{si\pm}$ and the remaining integrals $\mathcal{I}_{psi\pm}$ appearing in the source term (113).

Remarkably, expressions (112-113) show that the divergence of the axial current in fact corresponds to the first velocity moment of the kinetic equation for g^s_{0d}. Because a nontrivial CP-violation is tied to nontrivial complex phases in the *pseudoscalar*, or axial mass term, the fact that the source appears in the axial current nicely explains why the semiclassical source appears at first order in moment expansion in earlier semiclassical treatments [29, 15, 22].

The first source in (113) is to leading order in gradients spin-independent and does not violate CP. It is important for the phase transition dynamics however, in that it provides the dominant contribution to the friction on bubble walls from fermions [7, 9]. The second source is spin-dependent and CP-violating and it is thus responsible for baryogenesis. This is one of the main results of this paper, as it shows how the source from the semiclassical force enters to momentum integrated transport equations used in practical calculations. To promote equations (106) and (112) into transport equations for baryogenesis calculations

we still need to generalize them to include collisions as indicated in (46), which will be done elsewhere.

5.1 Spontaneous source in relaxation time approximation

We have so far shown how the source arising from the semiclassical force enters in the equations for currents, or equivalently momentum averaged transport equations. We shall now give a heuristic account on how sources have been modeled elsewhere in literature. The method very often used in EWBG considerations, apart from the WKB-computations, employs the relaxation time approximation for the kinetic equations. Here we discuss how the relaxation time approximation can be incorporated into our formalism, and then make a comparison with literature. Including collisions in the relaxation time approximation into the Liouville equation (72) results in the following kinetic equation for the distribution function f_q for a charge q:

$$\Big(\partial_t + \vec{v}_q \cdot \partial_{\vec{x}} + \vec{F}_q \cdot \partial_{\vec{k}}\Big) f_q = -\frac{f_q - f_{q0}}{\tau_q}\,, \tag{114}$$

where $\tau_q \equiv \Gamma_q^{-1}$ is the equilibration time for q (which we expect to be given by the relevant elastic scattering rate), $\vec{F}_q$ the semiclassical force and f_{q0} is the thermal equilibrium distribution function. In presence of a background field that violates q, one expects f_{q0} to be shifted with respect to the naive thermal equilibrium, leading to a 'spontaneous' source that violates q. This source is more important for thick walls when the equilibrium $f_q \approx f_{q0}$ is approximately attained on the wall. The *spontaneous baryogenesis* source was originally introduced by Cohen, Kaplan and Nelson [35, 16] in the context of two Higgs doublet models, and then subsequently refined to include the m^2-suppression in [41, 15]. The derivation was successively reconsidered in [36, 19, 20]. For example, in [20] the CP-violating vector current $j_q^0 = \int (dk_z/(2\pi)) f_{q0}$ for charginos in the MSSM was computed and inserted into the transport equations written in the relaxation-time approximation.

The spontaneous baryogenesis source can be in our formalism obtained simply by integrating (114) over the momenta. The source then becomes the CP-violating contribution to the vector current (44), which to first order in gradients (or equivalently $\hbar$) reads

$$n_{si} \equiv j^0_{si+} - j^0_{si-} \approx s|M_i|^2 \Theta_i' \int_{\omega \geq |M_i|} \frac{d\omega}{4\pi} \frac{1}{\sqrt{\omega^2 - |M_i|^2}} \frac{f_\omega}{\omega^2} \Big(1 + \frac{\omega}{T}(1 - f_\omega)\Big)\,, \tag{115}$$

where we approximated $f_{si\pm}$ by the equilibrium distribution function in plasma frame, $f_{si\pm} \to 1/(e^{\omega_{si\pm}/T}+1) \approx f_\omega \mp (s|M_i|^2\Theta_i'/2\omega^2) df_\omega/d\omega$, where $f_\omega = 1/(e^{\omega/T}+1)$, and we used $k_z dk_z = \omega d\omega$. To make a comparison with literature, note first that the spontaneous source (115) is *nonanalytic* in $|M_i|^2$. Since earlier attempts [36, 37, 17, 19] used expansions in powers of $|M_i|^2$ to compute the spontaneous source, their results are at best incomplete. Consider next the CP-violating source for charginos in the MSSM. According to our equation (82) it is given by $|M_d|^2\Theta' = \mathrm{diag}(m_+^2\Theta_+', m_-^2\Theta_-')$, where $m_\pm^2\Theta_\pm' = \mp(g^2/\Lambda)\Im(\mu m_2)(h_1 h_2)'$, showing the parametrical dependence $(h_1 h_2)'$ on the higgs fields. This is in contrast with Refs. [19, 20], where a source proportional to $h_1 h_2' - h_2 h_1'$ was found and claimed to be important for baryogenesis. The origin of the difference may be in the fact that we made use of the constraint equations, which is necessary to obtain the correct results.

Consider now the axial vector current. The corresponding spontaneous source can be easily obtained from (45):

$$s j^0_{5ds\pm} = \sum_{s_{k_z}=\pm} s_{k_z} \int \frac{d\omega}{8\pi} f_{sd\pm} = 0, \tag{116}$$

where we took $f_{si\pm} \to 1/(e^{\omega/T}+1)$. As a consequence, there is *no* spontaneous baryogenesis source from the axial vector current when computed in the relaxation time approximation.

An attempt to compute the spontaneous baryogenesis source was made by Riotto [37], where the divergence of the vector current was computed in the Schwinger-Keldysh formalism [42] and then, based on [36], identified with a spontaneous source [17]. In this way he found an equation which formally reads: $\partial_\mu j_q^\mu \sim$ spontaneous source. According to Eq (106) however *no* source appears in the continuity equation for the vector current. Instead sources appear in the continuity equation for the *axial* vector current (110-112), which has not been so far considered in literature.

6 Discussion and summary

The question of a first principle derivation of CP-violating fluxes in transport equations has been the main theoretical challenge of recent work on electroweak baryogenesis. In this paper we derive the kinetic equations appropriate for EWBG in a systematic gradient expansion

starting from the (Dirac) equation of motion for the two-point Wightman function $G^<$ in the collisionless limit. The gradient expansion we use is well controlled and corresponds to an expansion in the de Broglie wave length divided by the wall width. In EWBG applications one typically has $\ell_{dB}/\ell_w \ll 1$, so that such an expansion should be rapidly converging.

We have shown that to first order in $\hbar$ the collisionless kinetic equations for both fermions and bosons can be recast as the Liouville equations for a single particle distribution function where the group velocity and the semiclassical force terms contain all quantum information, and in particular the CP-violating terms which source baryogenesis. These results agree with Ref. [22], where the kinetic equations were obtained in the semiclassical WKB picture, originally developed for EWBG problem in [29, 15]. The outstanding contribution of this paper is in a first principles derivation of these results in a completely controlled approximation scheme. We also derive the semiclassical force in the general case of N mixing fermions and in particular for the chargino sector in both the MSSM and NMSSM. Finally we prove that there is no CP-violating force at first order in $\hbar$ for scalar fields (102). Let us point out that the fact that the quasiparticle picture of plasma still holds to order $\hbar$ in gradient expansion is not surprising since the gradient correction for fermions from a pseudoscalar (CP-violating) mass condensate can be equivalently reformulated in terms of a 'classical' axial vector field condensate [29, 15].

We have also studied the vector and axial vector current equations, and showed that, while the vector current is conserved, the axial current contains both CP-conserving and CP-violating sources. This is to be expected, as CP-violation in particular is known to be caused by complex pseudoscalar (axial) mass terms. We have then pointed out that the axial current equation corresponds to the first velocity moment of the kinetic equation. This explains why the CP-violating source appears at first order in moment expansion [29, 15]. We finally made connection between the present results and literature where the continuity equations were written in the relaxation time approximation. In this context the source has been claimed to appear either as the vector current divergence [37, 17] or the time-component of the vector current j^0 [36, 19, 20]. However, we have shown here that the vector current continuity equation (106) in fact contains no source. We have also computed j^0, and applied the result to the MSSM, and found a parametrically different result from Refs. [36, 19, 20].

For simplicity here we consider only the collisionless limit in 1+1 dimensions. One can show [27] that generalization to the 3+1 dimensional case does not affect our discussions in any qualitative way. The question of how to consistently include collisions we postpone to a future publication.

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References

[1] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B **155** (1985) 36.

[2] A. D. Sakharov, Pisma Zh. Eksp. Teor. Fiz. **5** (1967) 32 [JETP Lett. **5** (1967) 24].

[3] M. E. Shaposhnikov, JETP Lett. **44** (1986) 465 [Pisma Zh. Eksp. Teor. Fiz. **44** (1986) 364]; M. E. Shaposhnikov, Nucl. Phys. B **287** (1987) 757.

[4] K. Kajantie, M. Laine, K. Rummukainen and M. Shaposhnikov, Nucl. Phys. B **493** (1997) 413 [hep-lat/9612006]; Phys. Rev. Lett. **77** (1996) 2887 [hep-ph/9605288]; Nucl. Phys. B **466** (1996) 189 [hep-lat/9510020].

[5] M. Carena, M. Quiros and C. E. Wagner, Phys. Lett. B **380** (1996) 81 [hep-ph/9603420]; Nucl. Phys. B **524** (1998) 3 [hep-ph/9710401]; J.M. Cline and K. Kainulainen, Nucl. Phys. B **482** (1996) 73 [hep-ph9605235]; Nucl. Phys. B **510** (1998) 88 [hep-ph/9705201]; M. Laine, Nucl. Phys. B **481** (1996) 43 [Erratum-ibid. B **548** (1996) 637] [hep-ph/9605283]; M. Losada, Phys. Rev. D **56** (1997) 2893 [hep-ph/9605266]; D. Bödeker, P. John, M. Laine and M. G. Schmidt, Nucl. Phys. B **497** (1997) 387 [hep-ph/9612364]; M. Laine and K. Rummukainen, Nucl. Phys. B **535** (1998) 423 [hep-lat/9804019]; M. Laine and K. Rummukainen, Nucl. Phys. B **597** (2001) 23 [hep-lat/0009025].

[6] M. Pietroni, Nucl. Phys. B **402** (1993) 27 [hep-ph/9207227]; A. T. Davies, C. D. Froggatt and R. G. Moorhouse, Phys. Lett. B **372** (1996) 88 [hep-ph/9603388]; S. J. Huber and M. G. Schmidt, Eur. Phys. J. C **10** (1999) 473 [hep-ph/9809506].

[7] G. D. Moore and T. Prokopec, Phys. Rev. D **52** (1995) 7182 [hep-ph/9506475]; G. Moore and T. Prokopec, Phys. Rev. Lett. **75** (1995) 777 [hep-ph/9503296].

[8] J. M. Moreno, M. Quiros and M. Seco, Nucl. Phys. B **526** (1998) 489 [hep-ph/9801272]; J. M. Cline, G. D. Moore and G. Servant, Phys. Rev. D **60** (1999) 105035 [hep-ph/9902220].

[9] P. John and M. G. Schmidt, Nucl. Phys. B **598** (2001) 291 [hep-ph/0002050].

[10] G. D. Moore, JHEP **0003** (2000) 006 [hep-ph/0001274].

[11] J. M. Cline and G. D. Moore, Phys. Rev. Lett. **81** (1998) 3315 [hep-ph/9806354].

[12] A. G. Cohen, D. B. Kaplan and A. E. Nelson, Phys. Lett. B **245** (1990) 561.

[13] A. E. Nelson, D. B. Kaplan and A. G. Cohen, Nucl. Phys. B **373** (1992) 453.

[14] M. Joyce, T. Prokopec and N. Turok, Phys. Rev. D **53** (1996) 2930 [hep-ph/9410281].

[15] M. Joyce, T. Prokopec and N. Turok, Phys. Rev. D **53** (1996) 2958 [hep-ph/9410282].

[16] P. Huet and A. E. Nelson, Phys. Rev. D **53** (1996) 4578 [hep-ph/9506477].

[17] A. Riotto, Phys. Rev. D **53** (1996) 5834 [hep-ph/9510271]; Phys. Rev. D **58** (1998) 095009 [hep-ph/9803357].

[18] J. M. Cline, K. Kainulainen and A. P. Vischer, Phys. Rev. D **54** (1996) 2451 [hep-ph/9506284].

[19] M. Carena, M. Quiros, A. Riotto, I. Vilja and C. E. Wagner, Nucl. Phys. B **503** (1997) 387 [hep-ph/9702409].

[20] M. Carena, J. M. Moreno, M. Quiros, M. Seco and C. E. Wagner, Nucl. Phys. B **599** (2001) 158 [hep-ph/0011055].

[21] J. M. Cline, M. Joyce and K. Kainulainen, Phys. Lett. B **417** (1998) 79 [Erratum-ibid. B **448** (1998) 321] [hep-ph/9708393].

[22] J. M. Cline and K. Kainulainen, Phys. Rev. Lett. **85** (2000) 5519 [hep-ph/0002272]. J. M. Cline, M. Joyce and K. Kainulainen, JHEP **0007** (2000) 018 [hep-ph/0006119]. K. Kainulainen, Proceedings of COSMO 99, Trieste, Italy, Oct 1999, p. 383, World Scientific 2000 [hep-ph/0002273].

[23] S. J. Huber, P. John and M. G. Schmidt [hep-ph/0101249], Euro. Phys. J. C.

[24] S. J. Huber and M. G. Schmidt [hep-ph/0003122]; S. J. Huber and M. G. Schmidt, hep-ph/0011059.

[25] S. J. Huber, P. John, M. Laine and M. G. Schmidt, Phys. Lett. B **475** (2000) 104 [hep-ph/9912278].

[26] P. Huet and E. Sather, Phys. Rev. D **51** (1995) 379 [hep-ph/9404302].

[27] K. Kainulainen, T. Prokopec, M. G. Schmidt and S. Weinstock, in preparation.

[28] M. Joyce, K. Kainulainen and T. Prokopec, JHEP **0010** (2000) 029 [hep-ph/0002239].

[29] M. Joyce, T. Prokopec and N. Turok, Phys. Rev. Lett. **75** (1995) 1695 [Erratum-ibid. **75** (1995) 3375] [hep-ph/9408339].

[30] D. Bödeker, K. Kainulainen and T. Prokopec, in preparation.

[31] D. Vasak, M. Gyulassy and H. T. Elze, Annals Phys. **173** (1987) 462; H. T. Elze, M. Gyulassy and D. Vasak, Nucl. Phys. B **276** (1986) 706; I. Bialynicki-Birula, P. Gornicki and J. Rafelski, Phys. Rev. D **44** (1991) 1825.

[32] P. Zhuang and U. Heinz, Phys. Rev. D **53** (1996) 2096 [hep-ph/9510206].

[33] P. Zhuang and U. Heinz, Phys. Rev. D **57** (1998) 6525 [hep-ph/9801221].

[34] G. R. Shin and J. Rafelski, Annals Phys. **243** (1995) 65.

[35] A. G. Cohen, D. B. Kaplan and A. E. Nelson, Phys. Lett. B **263** (1991) 86.

[36] P. Huet and A. E. Nelson, Phys. Lett. B **355** (1995) 229 [hep-ph/9504427].

[37] A. Riotto, Nucl. Phys. B **518** (1998) 339 [hep-ph/9712221].

[38] J. Schwinger, J. Math. Phys. **2** (1961) 407; L. V. Keldysh, Zh. Eksp. Teor. Fiz. **47** (1964) 1515 [Sov. Phys. JETP **20** (1964) 1018].

[39] A. Pilaftsis and C. E. Wagner, Nucl. Phys. B **553** (1999) 3 [hep-ph/9902371].

[40] H. E. Haber and G. L. Kane, Phys. Rept. **117** (1985) 75.

[41] D. Comelli, M. Pietroni and A. Riotto, Phys. Lett. B **354** (1995) 91 [hep-ph/9504265].

[42] G.D. Mahan, Phys. Reps. **145** (1987) 251.

[43] M. Joyce, K. Kainulainen and T. Prokopec, Phys. Lett. B **468** (1999) 128 [hep-ph/9906411].